

How do we express transformations using matrix?

Find A.

$$1) A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$2) A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \quad A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$(x, y) \xrightarrow[k]{D} (kx, ky)$$

$$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} kx \\ ky \end{bmatrix}$$

Reflections

1) x-axis

$$(x, y) \longrightarrow (x, -y)$$

2) y-axis

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

3) $y = x$

$$(x, y) \longrightarrow (y, x)$$

Matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix}$$

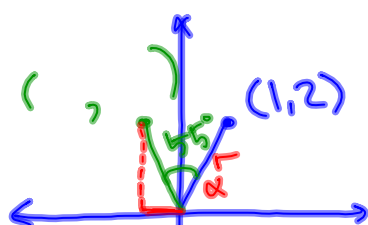
Rotations

Matrices

i) by 90°

$$(x, y) \rightarrow (-y, x) \quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x \end{bmatrix}$$

$$(1, 2) \xrightarrow{55^\circ, 0} (-1.06, 1.97)$$



$$\begin{aligned} r \cos \alpha &= 1 \\ r \sin \alpha &= 2 \end{aligned}$$

$$\begin{aligned} x &= r \cos(\alpha + 55^\circ) \\ &= r(\cos \alpha \cos 55^\circ - \sin \alpha \sin 55^\circ) \\ &= r \cos \alpha \cos 55^\circ - r \sin \alpha \sin 55^\circ \\ &= 1 \cos 55^\circ - 2 \sin 55^\circ \end{aligned}$$

$$\begin{aligned} y &= r \sin(\alpha + 55^\circ) \\ &= r(\sin \alpha \cos 55^\circ + \cos \alpha \sin 55^\circ) \\ &= r \sin \alpha \cos 55^\circ + r \cos \alpha \sin 55^\circ \\ &= 2 \cos 55^\circ + 1 \sin 55^\circ \end{aligned}$$

$$\begin{bmatrix} \cos 55^\circ & -\sin 55^\circ \\ \sin 55^\circ & \cos 55^\circ \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \cos 55^\circ - 2 \sin 55^\circ \\ 1 \sin 55^\circ + 2 \cos 55^\circ \end{bmatrix}$$